RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, MARCH 2021

THIRD YEAR [BATCH 2018-21]

PHYSICS (HONOURS)

Paper : VI [Gr. A]

Date : 18/03/2021

Time : 11 am - 1 pm

Full Marks : 50

Answer **any five** questions: [5 × 10]

1. a) A beam of X rays is scattered by electrons at rest. What is the energy of the X ray beam if the wavelength of the X rays scattered at 60° to the incident beam direction is 0.035 Å? [5]

   b) Given a metal whose work function is 4.05 eV. A radiation of wavelength $\lambda = 290\ nm$ falls on it. What is the stopping potential required to stop the most energetic emitted photoelectrons. [5]

2. a) Consider a wave function in the position space of the form

$$\psi(x) = Ae^{\mu x}, x < 0$$
$$= Ae^{-\mu x}, x > 0$$

   Determine the wave function in the momentum space $\phi(k)$. [5]

   b) Consider a particle with normalized wave function given by

$$\psi(x) = 2\alpha\sqrt{\alpha}xe^{-\alpha x} \quad x > 0$$
$$= 0 \qquad x < 0$$

   i) For what value of x does $P(x) = |\psi(x)|^2$ peaks? [1]

   ii) Calculate $\langle x \rangle$. [2]

   iii) What is the probability that the particle is found between $x = 0$ and $x = \frac{1}{\alpha}$. [2]

3. a) Consider a wave function of a particle $\psi(x) = A\sin kx$ in the region $0 < x < L$. Is k single valued or does it have multiple values. Comment. [2]

   b) Consider the momentum eigenfunction $\psi(x) = Ae^{ipx/\hbar}$. Is $\psi(x)$ also an eigenfunction of the position operator $\hat{x}$. Comment. [2]

   c) Consider the wave function of a harmonic oscillator $\Psi(x) = A(1 - 4\xi + 4\xi^2)e^{-\xi^2/2}$, which happens to be in a linear superposition of its first three eigenfunctions, $\psi_0(x) = \alpha e^{-\xi^2/2}$, $\psi_1(x) = \alpha\sqrt{2}\xi e^{-\xi^2/2}$ and $\psi_2(x) = \frac{\alpha}{\sqrt{2}}(2\xi^2 - 1)e^{-\xi^2/2}$. Determine the expectation value of the energy. $\xi = \sqrt{\frac{m\omega}{\hbar}}x$ as we had defined in the class. [6]

4. a) Consider a potential of the form

$$V(x) = \infty, x = 0$$
$$= 0, x > 0$$

   Will the energy eigenvalues be degenerate or non-degenerate. Explain. [2]

b) For a harmonic oscillator for which the raising and lowering operators are defined as $\hat{a}$ and $\hat{a}^\dagger$. Prove $\left[\hat{a}, \hat{a}^\dagger\right] = 1$. [3]

c) Consider a system in the superposition state

$$|\Psi\rangle = \frac{1}{\sqrt{19}}|\phi_1\rangle + \frac{2}{\sqrt{19}}|\phi_2\rangle + \sqrt{\frac{2}{19}}|\phi_3\rangle + \sqrt{\frac{3}{19}}|\phi_4\rangle + \sqrt{\frac{5}{19}}|\phi_5\rangle$$

Also $H|\phi_n\rangle = n\varepsilon_0|\phi_n\rangle$. Determine the expectation value of the energy $\langle H\rangle$. [5]

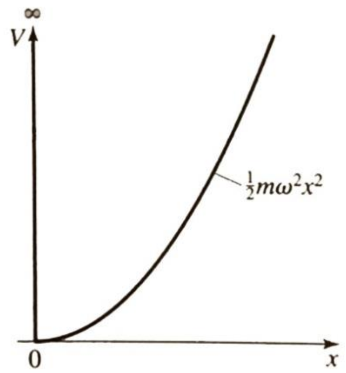
5. a) For a harmonic oscillator for which x is defined as $x = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a} + \hat{a}^\dagger\right)$ and the potential energy is $V = \frac{1}{2}m\omega^2 x^2$. Determine the expectation value of the potential energy $\langle V\rangle$. [5]

b) Consider a Gaussian wavepacket for a freely moving electron with

$$\omega = \frac{\hbar k^2}{2m}$$

If the initial width of the wave packet at $t = 0$ is $\Delta x(0) = \left(\frac{a}{2}\right) = 10^{-6}\, m$, determine the width of the wavepacket at a later time $\Delta x(t)$, at $t = 1$ seconds. Take the mass of the electron to be $0.9 \times 10^{-30}\, kg$. [5]

6. a) Sketch the first four eigenfunctions of a harmonic oscillator potential $V(x) = \frac{1}{2}m\omega^2 x^2$. Now consider a half harmonic oscillator potential (figure given below), using your knowledge of the properties of wave functions, sketch the first few eigenfunctions. Explain your answer. [5]



b) Prove [5]

i) $\left[x^n, p\right] = i\hbar n x^{n-1}$

ii) $\left[H, L^2\right] = 0$

7. a) For a particle in an infinite square well, show that the fractional difference in the energy between adjacent eigenvalues is [5]

$$\frac{\Delta E_n}{E_n} = \frac{2n+1}{n^2}$$

b) A hydrogen atom is in the state described by the wavefunction

$$\Psi = \frac{-i}{\sqrt{\pi a}\,4a^2} r e^{-r/2a} \sin\theta \sin\phi .$$

Determine the expectation value of the potential energy $\langle V(r) \rangle$. [5]

8. a) An electron inside a hydrogen atom is in the state described by the wave function

$$\Psi = \frac{1}{6}\left\{4\psi_{100} + 3\psi_{211} - \psi_{210} + \sqrt{10}\psi_{21-1}\right\}.$$

Compute the expectation values, $\langle E \rangle$ and $\langle L^2 \rangle$. [5]

b) A general spin state of an electron is given by

$$\chi = A\begin{pmatrix} 1-2i \\ 2 \end{pmatrix}$$

i) Determine the constant A. [1]

ii) If you measured S_z on this electron, what values would you get and what is the probability of each? [4]

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